Gravity Calculations from 3-D, Geologically Driven Models: A New Approach

SUMMARY

This work introduces a geologically based approach for calculating the gravity anomaly of any 3-dimensional (3-D) earth model with arbitrary density distribution. The calculation uses a 3-D modeling software package that facilitates model building using true geological operations such as deposition, erosion, faulting etc. Hence, models are not necessarily built from unnatural polyhedra, a shortcoming of many previous modeling schemes.

The gravity anomaly is computed by decomposing the earth model into a regular 3-D grid of finite cells, with each cell having a potentially different density. The routine simply sums the contribution of every cell at each point of observation. Currently, the computational scheme is best used within a broader modeling effort, rather than where potential field data are the primary modeling constraint. Future developments will make the process more interactive, and of greater utility in the early stages of modeling, or when additional data may be lacking.

ADVANTAGES

Some advantages of this approach over other gravity modeling schemes:

1) Calculations are based on precise, geologically grounded, fully 3-D earth models. While the limits of 2-D or 2.5-D models are self evident, even most 3-D modeling schemes rely on forward models built from arbitrary polygons of constant density. Building models from subsurface data using geologically representative processes (deposition, faulting etc.) yields models that are more accurate and intuitively appealing. The model building software is EarthVision® (Dynamic Graphics Inc.).

2) The earth models used in the calculation are an increasingly common product of the hydrocarbon exploration / development workflow. Based predominantly on seismic and well data, these precise 3-D models are often used for well-planning, volumetric calculations, and export to reservoir simulation. The models have also found widespread application in both large regional studies and localized site investigations typical of environmental assessments.

3) Density data are quickly and easily incorporated from well data which are likely already part of the earth model. Powerful gridding algorithms and geostatistics can be used to model the density data, yielding reliable property models even when sampling is sparse. Furthermore, gridding methods permit reconstruction to a pre-faulting configuration - thus the resulting properties can be true to their depositional (pre-faulting) arrangement. A cellular approach to defining the density model means no artificial constraints of single values of density for each unit, or even single
gradients. Literally any density distribution can be used because each cell / grid-node is considered separately.

4) Because model building and anomaly calculation are accomplished within a rich 3-D environment, users can harness the power of these visualization methods when evaluating the goodness of fit of the anomaly, and when making modifications to the model.

The advantages of this method prominently rely on embedding the scheme in a wider workflow of model building. The following section describes what could be a typical workflow for using this gravity calculation methodology.

**WORKFLOW**

To demonstrate the construction of the earth model, and subsequent calculation and visualization of a gravity response, we will use an example dataset from around a salt dome. The input data to this modeling effort consists of seismic data and well data (Figure 1). The seismic cube is depth migrated, so no depth conversion is necessary, although that step is routinely done within the modeling software for projects using time-migrated seismic.

![Figure 1. Input data used for model building. Seismic cube is depth migrated, approximately 5 km long per side. Uppermost reflector is the seabed, deformation caused by the nearly symmetrical salt dome is unmistakable. The well data are colored by density ranging from 2.0 to 3.1 g/cm³. The density values generally increase with depth.](image)

The seismic data were co-visualized with the well data for interpretation. The interpretation can be done either within EarthVision, or within another package from where the picks are exported. These seismic picks are the basis for building the earth model. First, the fault picks are quality controlled and limited by tip-line polygons in the case of dying faults (Figure 2). A ‘fault tree’ is then computed - either automatically or manually - describing the fault framework and fault dependencies.

Next, the stratigraphic picks from the seismic interpretation are gridded, with well-top picks optionally used as additional constraints on the shape and depth of the resulting horizon. These stratigraphic horizons are then superposed (as deposition or erosion) to form the stratigraphic sequence (Figure 2). During the gridding process, input horizon picks are automatically considered in the context of the modeled faults and other horizons. Hence the resulting solid earth model is geometrically consistent and faithful to the input data - even when input data are sparse.
**Figure 2:** Snapshot of the model building process. Colored planes constrain the extents of dying faults (nearly 100 faults were mapped above the top of salt). Mesh of yellow points shows individual depth picks interpreted from the seismic data. These picks will be gridded into a solid horizon surface. The solid green layer is the interpreted top of salt gridded in such a manner. These, and several other components, will be used to construct the final earth model. This type of 3-D visualization is extremely useful for quality control and interactive editing of the input data. (The model cube is roughly 5 km on each side.)

Once the geological model is built, it is populated with density data using 3-D grids within the stratigraphic layers. These 3-D grids consist of constants, gradients, or any arbitrary density distribution. Grid cell size is at the user’s discretion. 3-D gridding of density data uses reconstructive techniques to compensate for fault displacements. Furthermore, the density grids can be shaped conformal with layer boundaries leading to a better representation of the property distribution (Figure 3). For the purposes of the gravity algorithm, however, orthogonal grid lines are needed. Hence the density property is recomposed into a regular grid (Figure 3).

![Figure 3: Two types of the grid showing the density distribution. On the left is a conformal grid example wherein the grid lines of the property grid are shaped according to the bounding horizons. On the right is the same property model distributed in a regular grid. Currently the gravity algorithm requires this regular grid format as input. The salt and the upper zones within the model (not shown) were given constant densities.](image)

**THE GRAVITY ROUTINE**

Once the earth model is built and populated with density information, the calculation of the gravity anomaly is straightforward. The total anomaly at any point is simply the summation of the contributions of all the grid cells (cuboids) in the model (the contribution from a single cuboid is given below). An algorithm of this computation was given by Blakely (1995), and can be easily extended to cover any 3-D density distribution. Seber et al. (2001) presented a similar extension of
the Plouff calculation for computing gravity anomalies on a regional scale. One difference between
the case of Seber et al. and that presented here is that Seber et al. used only a small number (around
four) cells in the vertical dimension, and thus used cells very tall in relation to their horizontal
dimensions. The approach described herein more commonly uses grid cells that are similar in x, y,
and z dimensions to allow a fully 3-D density distribution (e.g., density changes / gradients are not
limited to the bounding horizons of stratigraphic units / polygons). Consequently there many more
cells used in the present method.

The contribution of a single cuboid, after Plouff (1976), is:

\[ g = G \rho \sum_{i=1,2} \sum_{j=1,2} \sum_{k=1,2} s [z_k \tan^{-1} \left( \frac{x_i y_j}{z_k R_{ijk}} \right) - x_i \ln(R_{ijk} + y_j) - y_j \ln(R_{ijk} + x_i)] \]

\( x_1 \) is the distance from the point of observation to the near edge of the cuboid along the x-axis, \( x_2 \) is
the distance to the far side of the cuboid, and \( y_{1,2} \) and \( z_{1,2} \) are similar measurements along the y and
z axes.

\( \rho = \) density
\( G = \) universal gravitational constant
\( R_{ijk} = \sqrt{x_i^2 + y_j^2 + z_k^2} \)
\( s = s_is_js_k; s_1 = -1; s_2 = 1 \)

Previously, elemental approaches such as this were rendered impractical due to prohibitively long
compute times. Modern computing power, however, allows small models to calculate nearly
instantly, and larger models usually complete in several minutes. (On a typical desktop machine,
the current algorithm can iterate through a single gravity computation - one cycle of the above
equation - many tens of thousands of times per second). Any delay is clearly unacceptable for
‘interactive’ modeling (where changes in the model are simultaneously reflected in changes to the
calculated anomaly). However, as stated above, the usage of this routine is currently intended as a
parallel process to the model building, rather than as an interactive tool. Clearly the user can
expedite the calculation through judicious gridding. For example, a single vertical grid cell can
represent layers in which no density variation exists, whereas detailed variations require higher
resolution grids.

Other inputs to the routine are the height at which to make the observations (currently a single
value across the entire model), any bulk shift, and the ‘background’ density. This ‘background’ is
the density of a hypothetical infinite horizontal slab, with thickness equal to the model height, in
which the model is ‘embedded’ before calculation. This simple device is used to lessen the edge
effects that are inevitable in a finite-sized model. Thus the algorithm only calculates relative
gravity anomalies, and not absolute gravity values. Observed values (that the calculated anomaly
will be compared against) will most likely be Bouguer anomalies or isostatic residuals.

The gravity calculation routine was tested using a variety of simple cases, such as an infinite slab,
small prisms, and other regular distributions. The anomalies were compared to those computed
directly from modification of the Newtonian formulae (e.g. Telford et al., 1990), or from publicly
available implementations of 2.5-D computations (e.g. Roecker, 1997). The results were found to
agree with the other methods. Notwithstanding the above, a key criteria in the approach discussed
herein is using grid resolutions high enough to capture the potential model complexities.
Figure 4: Gravity anomaly calculated from the density property grid. Color-filled contour lines are shown both floating above the solid earth model and draped on the model surface (upper layers of the model have been removed - the model is also cut to reveal internal layering). Displays such as this facilitate comparison of the anomaly with the model, make discrepancies easier to trace, and guide necessary alterations in the model. Contour spacing is 2 g.u.

Figure 4 hints at the power of 3-D visualization in comparing the calculated anomaly to the earth model. Furthermore, the measured gravity response could also be included in this display. These visualizations can greatly add to the understanding of the gravity response. Modifications to the model can then be made to better fit the observed and calculated anomalies. The power of the modeling building workflow makes such changes quick and easy.

APPLICATIONS

The above workflow demonstrates the usage of this algorithm on a scale typical to the hydrocarbon exploration industry. Another application where this routine could find utility is in regional models, where gravity data can form a more integral aspect of the modeling effort. Typically, these regional models are in areas of measured gravity anomalies, but the gravity signature has often only been modeled along 2-D profiles. A good example of this is the USGS Hayward model (Ponce et al. 2003). In this developing 3-D regional model, the San Leandro gabbro is well expressed by potential field methods, and is seen as a controlling factor on the seismicity along the Hayward fault zone. Modeling this entire region as a unified 3-D model could assist in the delimitation of the extents and nature of the gabbro. At another scale, this method could be equally useful for small site investigations by shallow geophysical work. Such a model might include void/fault detection in the framework of a larger hydrogeologic investigation.

CONCLUSIONS

Presented herein is a method to obtain the gravity signature from any given geological model. The fully 3-D, geologically driven earth model is decomposed into a 3-D grid and the gravity signature is calculated iteratively. Currently this system is best used as part of a complete model-building workflow, in which the gravity calculation is one of many elements in the modeling process.
Future developments of the method aim to make it more suitable for cases where the gravity signature is the primary input data. Such developments could include automated inversion and interactive modeling - although much further work is required to make this a reality. A key issue is the need for additional calculation speed. This could be achieved through code optimization, parallel processing techniques, and automated aggregation of same-density grid cells to reduce the total number of calculations. Inclusion of magnetic calculations is another logical development. In the meantime, this approach represents an interesting extension and application of the EarthVision modeling software that avoids some drawbacks of typical gravity modeling.

REFERENCES


